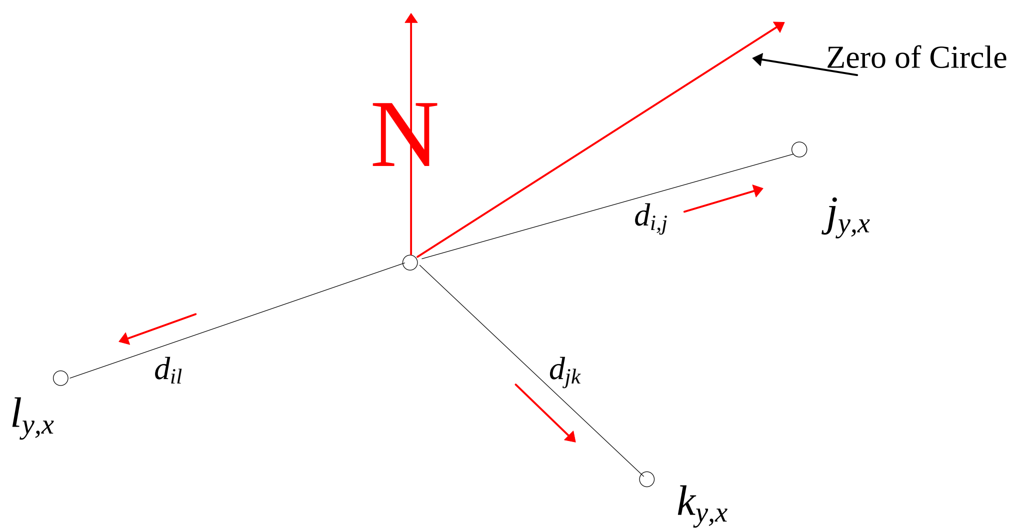
Mathematical Model for a Set of Directions on a Conformal Mapping Plane

Dr. Peter A. Steeves, P. Eng.
Geodetic Software Systems

When a set of directions are observed, the zero position of the theodolite horizontal plate is pointed, arbitrarily, in an unknown direction. If the orientation of the zero position would be known, an observed direction would be an observed azimuth. We may think of directions as being azimuths, made up of two parts; an observed quanity to be adjusted and an unknown quantity, orientation unknown, to be estimated. The orientation unknown, Z, can be approximated by some quantity, $\tilde{Z}$, where $Z = \tilde{Z} + \Delta_z$ and Δ_z is estimated by the least squares solution. As a matter of convenience, a set of directions is numerically rotated so that the zero position of the plate is mathematically orientated to one of the lines of sight. The approximate value of the orientation unknown can now be set to equal the approximate azimuth of the chosen line, computed from the approximate coordinates of the end points.



$$AZ_{ij} = \alpha_{ij} = \tan^{-1}\left\{\frac{x_j - x_i}{y_j - y_i}\right\}$$

In the Figure we have a set of three directions.

$$F_1(\tilde{\Delta}) \quad = \quad \tilde{\alpha}_{ij} - \tilde{Z} \quad = \quad \tan^{-1}\left\{ \frac{\tilde{x}_j - \tilde{x}_i}{\tilde{y}_j - \tilde{y}_i} \right\} \quad - \quad \tilde{Z}$$

$$F_2(\tilde{\Delta}) \quad = \quad \tilde{\alpha}_{ik} - \tilde{Z} \quad = \quad \tan^{-1}\left\{ \frac{\tilde{x}_k - \tilde{x}_i}{\tilde{y}_k - \tilde{y}_i} \right\} \quad - \quad \tilde{Z}$$

$$F_3(\tilde{\Delta}) \quad = \quad \tilde{\alpha}_{il} - \tilde{Z} \quad = \quad \tan^{-1}\left\{ \frac{\tilde{x}_l - \tilde{x}_i}{\tilde{y}_l - \tilde{y}_i} \right\} \quad - \quad \tilde{Z}$$

$$A \quad = \quad \left. \frac{\partial F}{\partial \Delta} \right|_{\tilde{\Delta}} \quad = \quad \left[\frac{\partial F}{\partial y_i}, \quad \frac{\partial F}{\partial x_i}, \quad \frac{\partial F}{\partial y_j}, \quad \frac{\partial F}{\partial x_j}, \quad \frac{\partial F}{\partial y_k}, \quad \frac{\partial F}{\partial x_k}, \quad \frac{\partial F}{\partial y_l}, \quad \frac{\partial F}{\partial x_l}, \quad \frac{\partial F}{\partial z} \right]$$

$$A \quad = \quad \left[\begin{array}{cccccc} \frac{+(\tilde{x}_j - \tilde{x}_i)}{\tilde{S}_{ij}^{\,2}}, & \frac{-(\tilde{y}_j - \tilde{y}_i)}{\tilde{S}_{ij}^{\,2}}, & \frac{+(\tilde{x}_j - \tilde{x}_i)}{\tilde{S}_{ij}^{\,2}}, & \frac{-(\tilde{y}_j - \tilde{y}_i)}{\tilde{S}_{ij}^{\,2}}, & 0, & 0, \\ \frac{+(\tilde{x}_k - \tilde{x}_i)}{\tilde{S}_{ik}^{\,2}}, & \frac{-(\tilde{y}_k - \tilde{y}_i)}{\tilde{S}_{ik}^{\,2}}, & 0, & 0, & \frac{+(\tilde{x}_k - \tilde{x}_i)}{\tilde{S}_{ik}^{\,2}}, & \frac{-(\tilde{y}_k - \tilde{y}_i)}{\tilde{S}_{ik}^{\,2}}, \\ \frac{+(\tilde{x}_l - \tilde{x}_i)}{\tilde{S}_{il}^{\,2}}, & \frac{-(\tilde{y}_l - \tilde{y}_i)}{\tilde{S}_{il}^{\,2}}, & 0, & 0, & 0, & 0, \end{array} \right\|$$

$$\left. \begin{array}{ccc} 0, & 0, & -1 \\ 0, & 0, & -1 \\ \frac{+(\tilde{x}_l - \tilde{x}_i)}{\tilde{S}_{il}^{\,2}}, & \frac{+(\tilde{y}_l - \tilde{y}_i)}{\tilde{S}_{il}^{\,2}}, & -1 \end{array} \right]$$

$$W = \begin{bmatrix} \tilde{\alpha}_{ij} & - & \tilde{Z} & - & \left(d_{ij} - d_{ij}\right) \\ \tilde{\alpha}_{ik} & - & \tilde{Z} & - & \left(d_{ik} - d_{ij}\right) \\ \tilde{\alpha}_{il} & - & \tilde{Z} & - & \left(d_{il} - d_{ij}\right) \end{bmatrix}$$

but d_{ij} has been mathematically rotated to zero

$$W = \begin{bmatrix} \tilde{\alpha}_{ij} & - & \tilde{Z} & & \\ \tilde{\alpha}_{ik} & - & \tilde{Z} & - & d_{ik} \\ \tilde{\alpha}_{il} & - & \tilde{Z} & - & d_{il} \end{bmatrix}$$

$\tilde{Z}$ is set equal to $\tilde{\alpha}_{ij}$; then,

$$W = \begin{bmatrix} 0.0 & & & & \\ \tilde{\alpha}_{ik} & - & \tilde{\alpha}_{ij} & - & d_{ik} \\ \tilde{\alpha}_{il} & - & \tilde{\alpha}_{ij} & - & d_{il} \end{bmatrix}$$