Mathematical Model for an Observed Distance on a Conformal Mapping Plane

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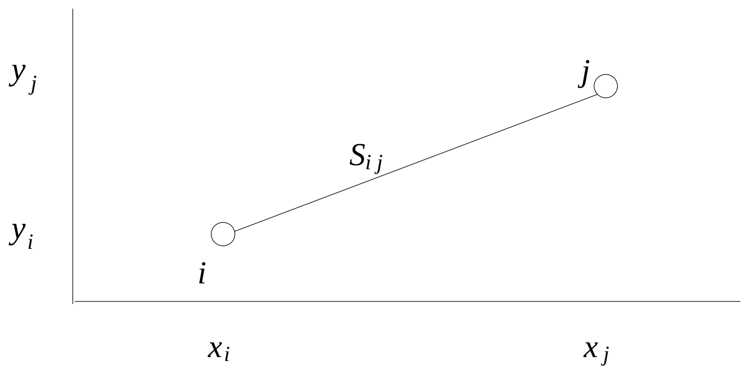
<u>The mathematical model for a horizontal distance:</u>



From basic trigonometry we have:

$$S_{ij} = \left\{ \left(y_j - y_i \right)^2 + \left(x_j - x_i \right)^2 \right\}^{1/2}$$

a non-linear equation. Non-linear equations are not easy to model in a least squares adjustment, therefore, we shall substitute the rigorous mathematical model with the first terms (linear terms) of the Taylor's series expansion of the mathematical model and iterate the solution to convergence.

$$F(\tilde{\Delta}) = \tilde{S}_{ij} = \left\{ \tilde{y}_j^2 - 2\,\tilde{y}_j\,\tilde{y}_i + \tilde{y}_i^2 + \tilde{x}_j^2 - 2\,\tilde{x}_j\,\tilde{x}_i + \tilde{x}_i^2 \right\}^{1/2}$$

$$A = \left. \frac{\partial F}{\partial \Delta} \right|_{\tilde{\Delta}} = \left[\frac{\partial F}{\partial y_i}, \ \frac{\partial F}{\partial x_i}, \ \frac{\partial F}{\partial y_j}, \ \frac{\partial F}{\partial x_j} \right]$$

$$\frac{\partial F}{\partial y_i} = \frac{-2\left(\tilde{y}_j - \tilde{y}_i \right)}{+2\,\tilde{S}_{ij}} = \frac{-\left(\tilde{y}_j - \tilde{y}_i \right)}{\tilde{S}_{ij}}$$

$$\frac{\partial F}{\partial y_j} = \frac{+2\left(\tilde{y}_j - \tilde{y}_i\right)}{+2\tilde{S}_{ij}} = \frac{+\left(\tilde{y}_j - \tilde{y}_i\right)}{\tilde{S}_{ij}}$$

$$\frac{\partial F}{\partial x_i} = \frac{-2\left(\tilde{x}_j - \tilde{x}_i\right)}{+2\tilde{S}_{ij}} = \frac{-\left(\tilde{x}_j - \tilde{x}_i\right)}{\tilde{S}_{ij}}$$

$$\frac{\partial F}{\partial x_j} = \frac{+2\left(\tilde{x}_j - \tilde{x}_i\right)}{+2\tilde{S}_{ij}} = \frac{+\left(\tilde{x}_j - \tilde{x}_i\right)}{\tilde{S}_{ij}}$$

$$A = \left[\frac{-\left(\tilde{y}_j - \tilde{y}_i\right)}{\tilde{S}_{ij}}, \quad \frac{-\left(\tilde{x}_j - \tilde{x}_i\right)}{\tilde{S}_{ij}}, \quad \frac{+\left(\tilde{y}_j - \tilde{y}_i\right)}{\tilde{S}_{ij}}, \quad \frac{+\left(\tilde{x}_j - \tilde{x}_i\right)}{\tilde{S}_{ij}} \right]$$

A is known as the design matrix for the unknown parameters.

$$\hat{X} = \begin{bmatrix} dy_i \\ dx_i \\ dy_j \\ dx_j \end{bmatrix}$$

$$W = \tilde{S}_{ij} - S_{ij} = F\left(\tilde{\Delta}\right) - S \quad (observed)$$

The design matrix for the observations is:

$$B = \left.\frac{\partial F}{\partial L}\right|_{L} = \begin{bmatrix} -1 & 0 & . & . & . & . & 0 & 0 \\ 0 & -1 & . & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & . & . & -1 & 0 \\ 0 & 0 & . & . & . & . & 0 & -1 \end{bmatrix}$$

a negative Identity matrix, therefore, a least squares adjustment of distances uses the parametric case.