MULTIFACTORIAL MULTIPLICATION
(Solving Problems that Contain Two or More Multipliers)

ABACUS: MYSTERY OF THE BEAD
The Bead Unbaffled - An Abacus Manual

Multifactorial Multiplication

The following is a very powerful technique for solving multiplication problems that contain two or more multipliers. When explaining the technique I'll use standard terminology. For example in the problem 1 x 2 x 3 = 6; the number 1 is the multiplicand, both 2 and 3 are the multipliers and 6 is the product.

When using the *standard method* to solve problems of multiplication on a soroban, product answers are routinely placed immediately to the right of the multiplicand. The standard method remains very efficient but sometimes there can be a little confusion when a multiplication problem contains two or more multipliers. Because the position of the unit number in a product moves to the right with each each operation, keeping track of the unit number can be problematic. Especially when there is a successive number of multiplications. This technique offers a very good solution.

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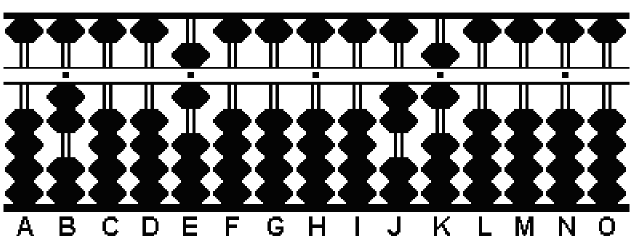
Setting Numbers onto the Soroban

When solving problems using the standard Japanese method, the operator clears the multiplicand after each multiplication step. In this technique products are actually added to the multiplicand. In other words the multiplicand becomes a part of the product. As a result, when setting multipliers onto the soroban the value of each should be reduced by 1.

For example in 26 x 7 x 3 = 546, when setting numbers onto the soroban the multipliers 7 and 3 should each be reduced by 1 and placed onto the soroban as 6 and 2.

Example: 26 x 7 x 3 = 546

Step 1: Designate rod K to be the unit rod. Set the multiplicand 26 on unit rods JK. Set the first multiplier 6 (7-1) on rod E and the second multiplier 2 (3-1) on rod B. (Fig.1)



(Fig.1)

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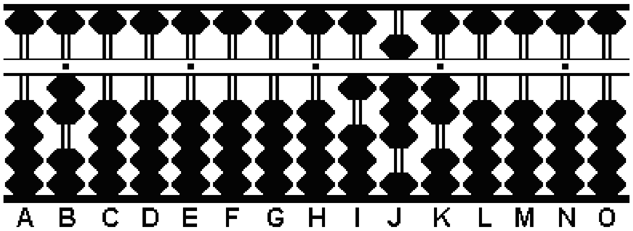
Step 1

A B C D E F G H I J K L M N O
  .     .     .     .     .
0 2 0 0 6 0 0 0 0 2 6 0 0 0 0   Step 1
```

Step 2: Multiply 2 on rod J by the 6 on E and add the product 12 to rods IJ.
2a: Multiply 6 on on rod K by 6 on E. Add the product 36 to rods JK.
2b: Having finished with the 6 on E it can be cleared from the frame. This leaves a partial product of 182 on rods IJK. (Fig.2)

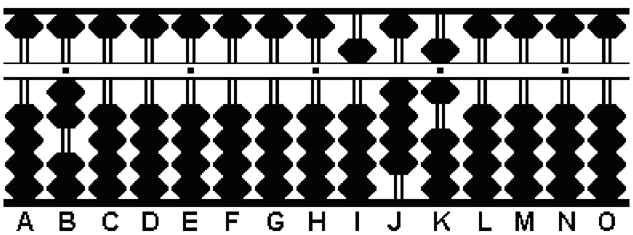


(Fig.2)

```
Step 2

A B C D E F G H I J K L M N O
  .     .     .     .     .
0 2 0 0 6 0 0 0 0 2 6 0 0 0 0
              + 1 2              Step 2
0 2 0 0 6 0 0 0 1 4 6 0 0 0 0
                + 3 6            Step 2a
0 2 0 0 6 0 0 0 1 8 2 0 0 0 0
       (-6)                      Step 2b
0 2 0 0 0 0 0 0 1 8 2 0 0 0 0
```

Step 3: Multiply 1 on rod I by 2 on rod B and add the product 02 to rods HI.
3a: Multiply 8 on rod J by 2 on B. Add the product 16 to rods IJ.
3b & the answer: Multiply 2 on rod K by 2 on B and add the product 04 to rods JK, leaving the answer 546 on rods IJK. (Fig.3)



(Fig.3)

```
Step 3

A B C D E F G H I J K L M N O
  .     .     .     .     .
0 2 0 0 0 0 0 0 1 8 2 0 0 0 0
            + 0 2                Step 3
0 2 0 0 0 0 0 0 3 8 2 0 0 0 0
              + 1 6              Step 3a
0 2 0 0 0 0 0 0 5 4 2 0 0 0 0
                + 0 4            Step 3b
0 2 0 0 0 0 0 0 5 4 6 0 0 0 0
```

Predetermining the Unit Rod

With this technique, problems that involve multiplying decimal numbers are very easy to deal with. The rule is simple: **Designate a unit rod. Then, for every decimal number in the multiplier(s), shift the multiplicand one rod to the right.** (Predetermining the unit rod, an *alternative*.)

Example: 2.6 x 0.017 x 1.4 = 0.06188

In this example notice the multipliers. The first multiplier 0.017 has three decimal numbers. The second multiplier 1.4 has one decimal number for a total of 4 decimal numbers. In order to predetermine the unit rod this must be taken into account when setting the multiplicand onto the soroban.

Step 1: Choose rod H to be the unit rod. Normally you'd think that the multiplicand 2.6 would be set on rods HI. However, the two multipliers have a total of 4 decimal numbers. Therefore, from unit rod H count 4 rods to the right. Set 26 on rods LM. This will allow the unit number in the product to fall naturally on designated unit rod H. Since we've already made allowance for the decimal numbers, setting the multipliers will be very easy. Just set 16 (17 -1) on rods FG and 13 (14 -1) on rods BC. (Fig.4)

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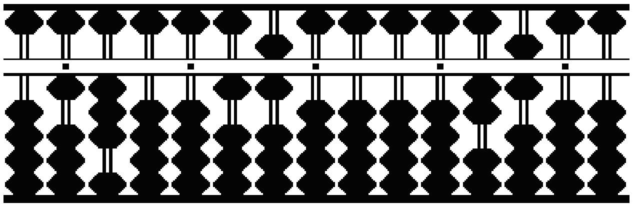
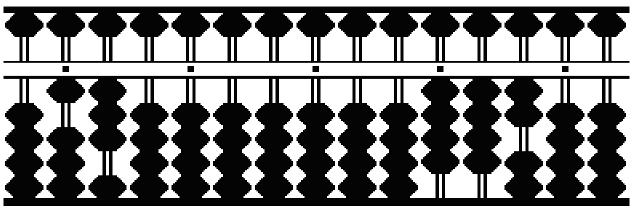


Step 1

```
A B C D E F G H I J K L M N O
  .     .     .     .     .
0 1 3 0 0 1 6 0 0 0 0 2 6 0 0   Step 1
```

Step 2: Multiply 2 on rod L by 1 on F and add the product 02 to rods JK.
2a: Multiply 2 on rod L by 6 on G and add the product 12 to rods KL.
2b: Multiply 6 on rod M by 1 on F and add the product 06 to rods KL.
2c: Multiply 6 on rod M by 6 on G and add the product 36 to rods LM.
2d: Having finished with the multiplier 16 on rods FG, it can be cleared from the frame. This leaves a partial product of 442 on rods KLM. (Fig.5)

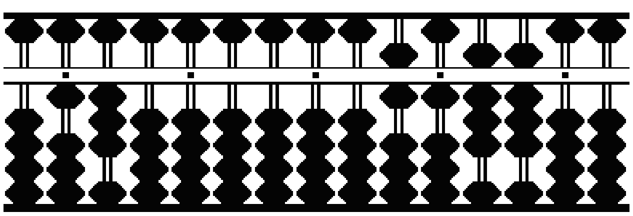


(Fig.5)

Step 2

```
A B C D E F G H I J K L M N O
  .     .     .     .     .
0 1 3 0 0 1 6 0 0 0 0 2 6 0 0
                  + 0 2           Step 2
                    + 1 2         Step 2a
0 1 3 0 0 1 6 0 0 0 3 4 6 0 0
                    + 0 6         Step 2b
                      + 3 6       Step 2c
0 1 3 0 0 1 6 0 0 0 4 4 2 0 0
        (-1 6)                    Step 2d
0 1 3 0 0 0 0 0 0 0 4 4 2 0 0
```

Step 3: Multiply 4 on rod K by 1 on B. Add the product 04 to rods IJ.
3a: Multiply 4 on rod K by 3 on C and add the product 12 to rods JK.
3b: Multiply 4 on rod L by 1 on B. Add the product 04 to rods JK.
3c: Multiply 4 on rod L by 3 on C and add the product 12 to rods KL.
3d: Multiply 2 on rod M by 1 on B. Add the product 02 to rods KL.
3e & the answer: Multiply 2 on rod M by 3 on C and add the product 06 to rods LM. This leaves the answer 6188 to rods JKLM. Because rod H is the designated unit rod the answer actually reads 0.06188



(Fig.6)

Step 3

```
A B C D E F G H I J K L M N O
  .     .     .     .     .
0 1 3 0 0 0 0 0 0 0 4 4 2 0 0
                + 0 4             Step 3
                  + 1 2           Step 3a
0 1 3 0 0 0 0 0 0 5 6 4 2 0 0
                  + 0 4           Step 3b
                    + 1 2         Step 3c
0 1 3 0 0 0 0 0 0 6 1 6 2 0 0
                    + 0 2         Step 3d
                      + 0 6       Step 3e
0 1 3 0 0 0 0 0 0 6 1 8 8 0 0
```

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A Simple 12x Multiplication Exercise

Here we'll take a slightly different turn. Let's use the multifactorial method again but instead of multiplying by two multipliers, we'll multiply by only one. Multiplying by 12 is quite a common operation. Using the multifactorial method, it's very quick and easy to solve problems that involve multiplying by 12.

Example 143 x 12 = 1716

Step 1: Designate rod K to be the unit rod. Set the multiplicand 143 on rods IJK. Set the multiplier 11 (12-1) on rods DE. (Fig.7)

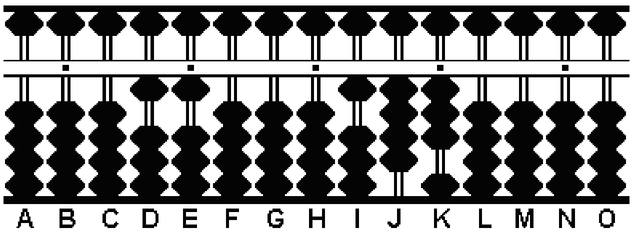

Fig.7

```
Step 1

A B C D E F G H I J K L M N O
___.___.___.___.___.___.___
0 0 0 1 1 0 0 0 1 4 3 0 0 0 0   Step 1
```

Step 2: Multiply 1 on rod I by the 11 on DE and add the product 11 to rods HI.

2a: Multiply 4 on on rod J by 11 on DE. Add the product 44 to rods IJ.

2b & the answer: Multiply 3 on on rod K by 11 on DE. Add the product 33 to rods JK. This leaves the answer 1716 on rods HIJK. (Fig.8)

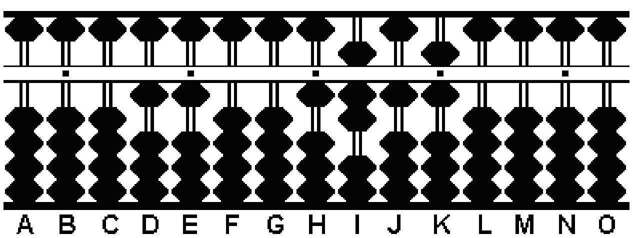

Fig.8

```
Step 2

A B C D E F G H I J K L M N O
___.___.___.___.___.___.___
0 0 0 1 1 0 0 0 1 4 3 0 0 0 0
              + 1 1              Step 2
_____________________________
0 0 0 1 1 0 0 1 2 4 3 0 0 0 0
                + 4 4            Step 2a
_____________________________
0 0 0 1 1 0 0 1 6 8 3 0 0 0 0
                  + 3 3          Step 2b
_____________________________
0 0 0 1 1 0 0 1 7 1 6 0 0 0 0
```

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Real Life Situations

1) Let's say a customer buys 27 note books at a cost of $1.25 each and has to pay a tax of 7%. In order to determine the total cost, the calculation would be 27 x 1.25 x 1.07 = 36.1125.

2) The area of a triangle can be found by multiplying one-half the base times the height. (Note: to find ½ of a number it can be multiplied by 0.5) If a triangle has a base length of 12 inches and a height of 7.5 inches, its area is 0.5 x 12 x 7.5 = 45 inches.

3) What is the volume and surface area of a cube having a side-length of 3.1 cm?
Its volume would be 3.1 × 3.1 × 3.1 = 29.791 cubic centimeters.
Its surface area would be 6 × 3.1 × 3.1 = 57.66 square centimeters.

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