



FIND THE SQUARE ROOT OF 2 TO SEVEN DECIMAL PLACES

I've often wished that Professor Kato had gone a little further and illustrated his technique using one or two examples where he would take a large decimal answer from a whole number. Here's a Square Root problem that's a good example. Because of its long decimal answer, we get a pretty good feel for the technique and for how numbers should be placed on the soroban.

[Kato's methods](#) for finding the Square Root of a number have been well documented. Please study them first. With the exception of the first few steps, this example will only show images of the soroban after completing each of the operations. The diagrams can be used as a guideline to help assess one's understanding of the technique.

A Couple of things to Remember:

i) In the following it's important to remember that where possible the numbers making up the radicand are always divided into groups of two. For example when finding the Square Root of 2, the 2 stands alone but subsequent groups should be paired as 00 00 00 and so on.

ii) Prof. Kato shows us how to subtract $\frac{1}{2}$ of a squared number ([main page](#)). As a result, sometimes fractions like 0.5, 4.5, 12.5, etc. will come into play. Remember, because it's a fraction, when subtracting 0.5 the operation will always carry another 0.5 over into the next pairing of numbers (see steps 1b and 3b for clarification).

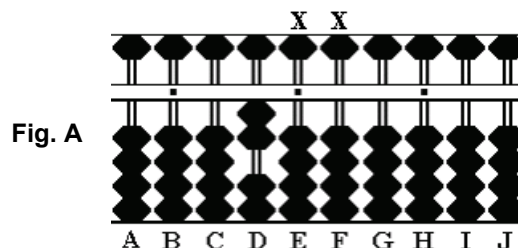
To illustrate I'll do the first few steps of finding the square root of 2.

In this step the group is 2 on rod D with the next pair 00 (XX) on rods EF.

Step 1:

Find the square root of 2.

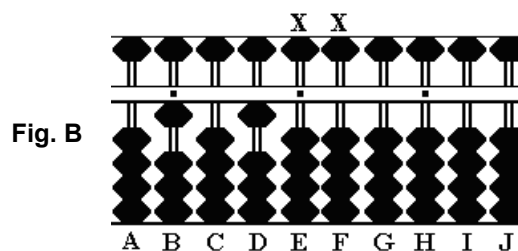
Set 2 on rod D. The group at this point is 2 on rod D and the pair 00 on rods EF. (Fig. A)



Step 1a:

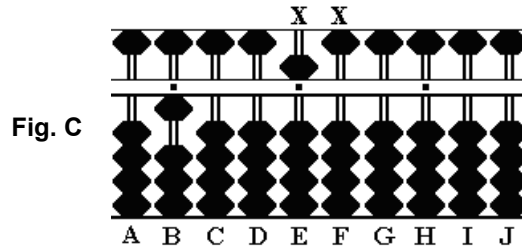
Find the largest perfect square that is less than or equal to 2. The answer is 1. Place 1 on rod. B

Square 1 on B ($1 \times 1 = 1$), subtract 1 from 2 on rod D leaving 1. (Fig. B)



Step 1b:

Subtract *half* of 1 from rod D. This carries .5 over to Rod E in the next pairing. (Fig. C)



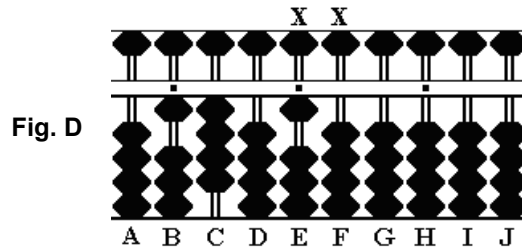
In each of the division steps below there must be enough remainder to subtract *half* the answer squared. Watch this next step.

Step 2:

Divide 1 on rod B into 5 on rod E.

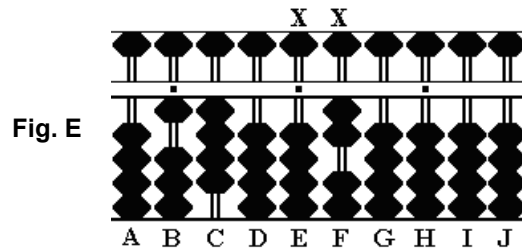
It looks like the answer should be 5, but there would be no remainder. Choose 4 instead. The resulting remainder allows us to complete **Step 2a** (see below).

Place the answer 4 on rod C. Multiply 4 on C by 1 on B and subtract the result from 5 on rod E. This leaves 1. (Fig. D)



Step 2a:

Square 4 on rod C ($4 \times 4 = 16$) and subtract *half* (8) from rods EF. This leaves the partial answer 1.4 on rods BC and a remainder 2 on rod F. (Fig. E)

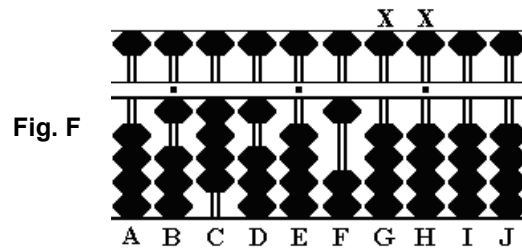


In this step add another pairing of 00 (XX), this time to rods GH.

Step 3:

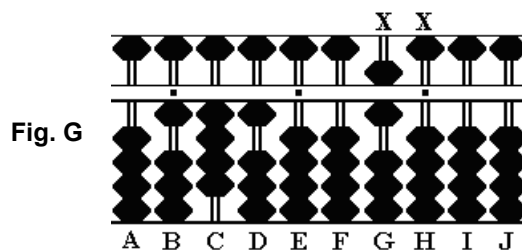
Divide 1 on B into 2 on F and place the answer 1 on rod D.

Multiply 1 on rod D by 1 on B ($1 \times 1 = 1$). Subtract 1 from 2 on Rod F leaving 1. (Fig. F)



Step 3a:

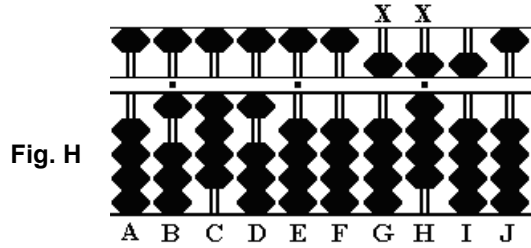
Multiply 1 on rod D by 4 on C ($1 \times 4 = 4$). Subtract 4 from rods FG. This leaves 6 on rod G. (Fig. G)



Step 3b:

Square 1 on rod D ($1 \times 1 = 1$) and subtract *half* (.5) from the remainder. This leaves the partial answer 1.41 on rods BCD and the remainder 595 on rods GHI. (Fig. H)

**When subtracting .5 (not a whole number) from rod H it adds .5 to the next pairing of numbers, in this case rod I. **

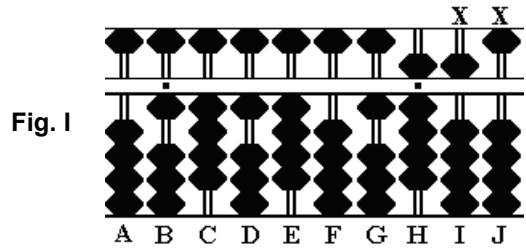


In this step add another pairing of 00 (XX), this time to rods IJ.

Step 4:

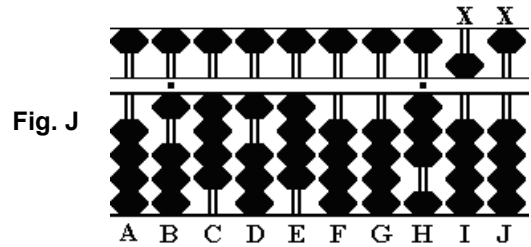
Divide 1 on rod B into 5 on rod G.

Place the answer 4 on rod E. Multiply 4 on E by 1 on B and subtract the result from rod G. (Fig. I)



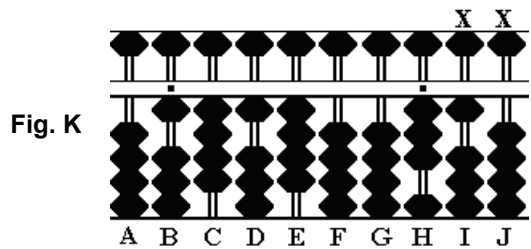
Step 4a:

Multiply 4 on E by 4 on C and subtract 16 from rods GH. (Fig. J)



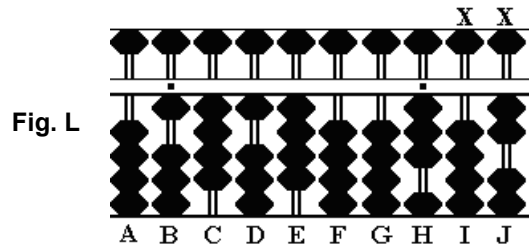
Step 4b:

Multiply 4 on E by 1 on D and subtract 04 from rods HI. (Fig. K)



Step 4c:

Square 4 on rod E ($4 \times 4 = 16$) and subtract *half* (8) from rods IJ. This leaves the partial answer 1.414 on rods BCDE and the remainder 302 on rods JIJ. (Fig. L)

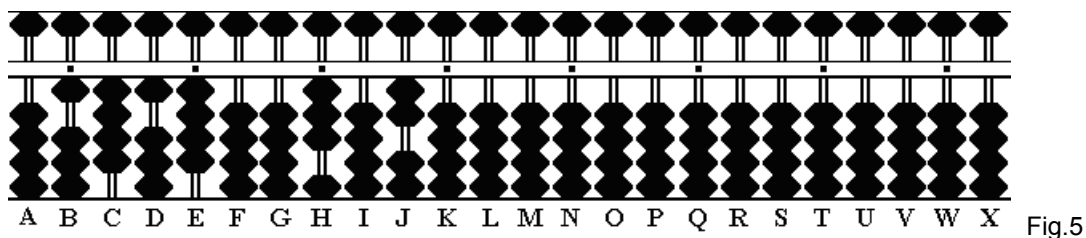
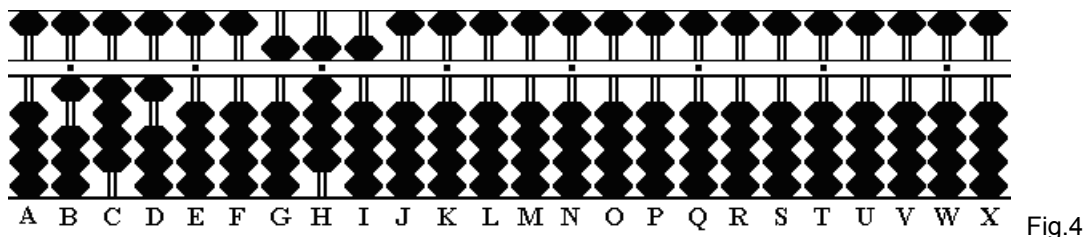
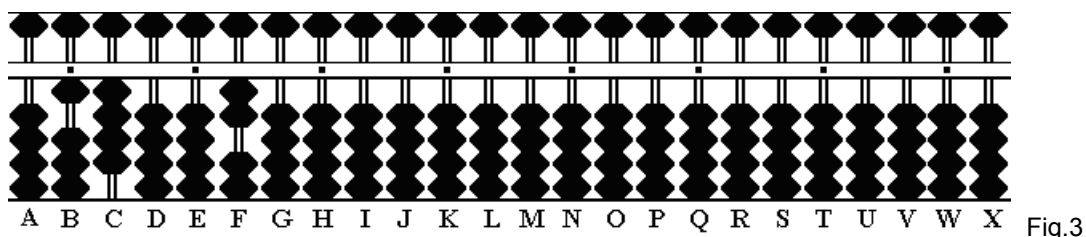
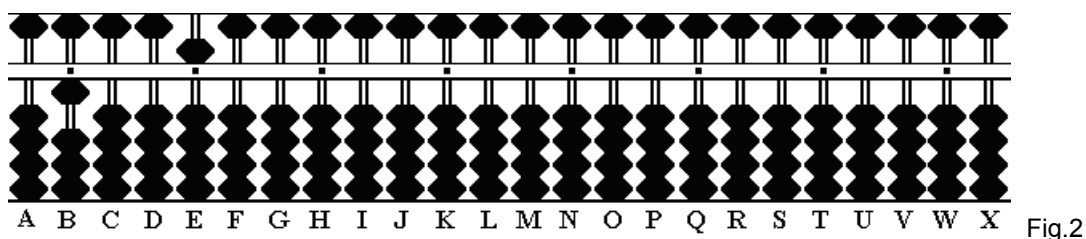
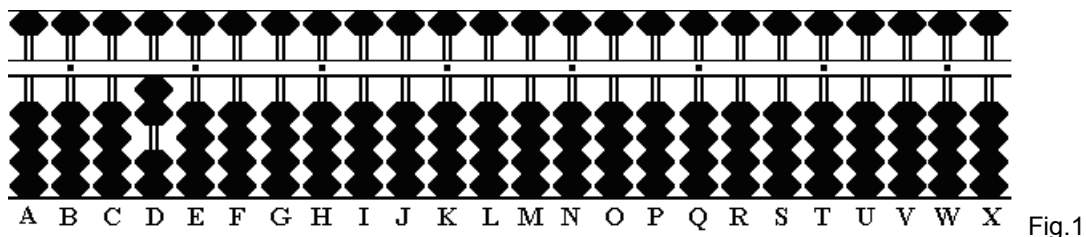


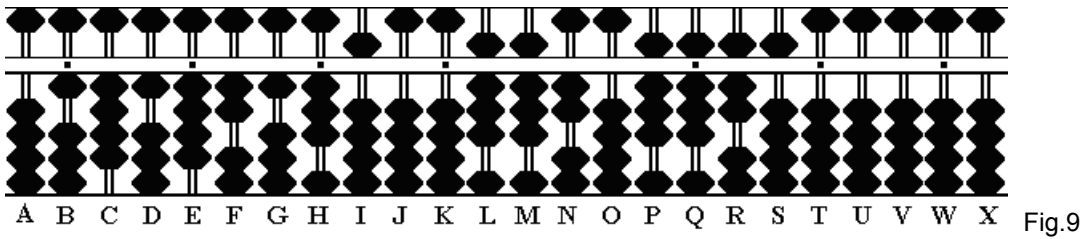
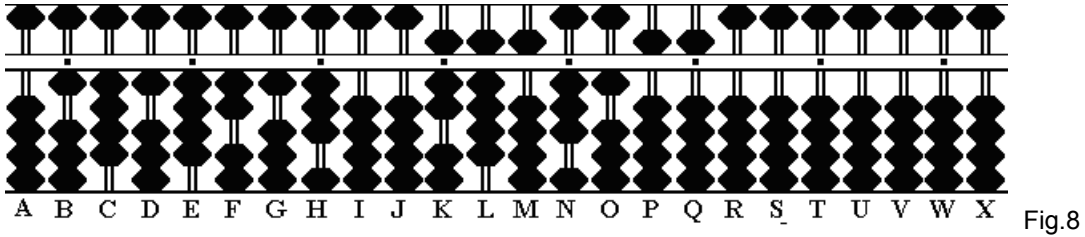
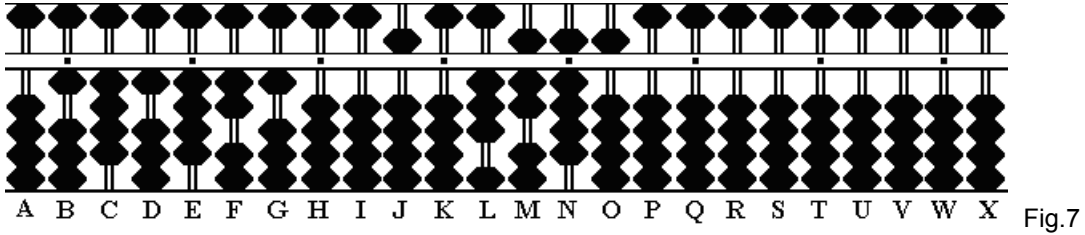
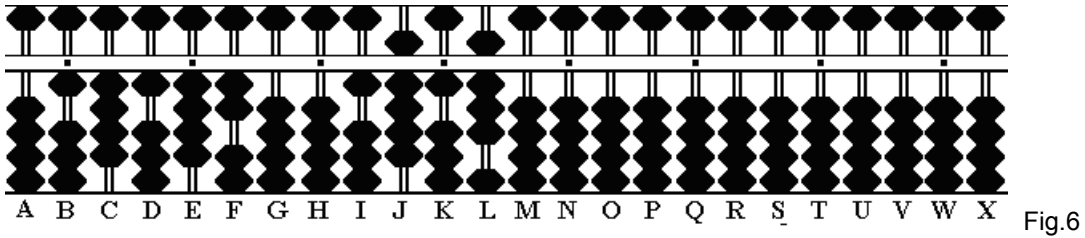
*At this point the problem has been solved to three decimal places giving a pretty good idea of how the technique works. In order to continue just add further pairs of 00 (XX), complete the division and subtract *half* the squared number as shown above.*

The following diagrams take the example to seven decimal places. The process starts from the beginning and shows the soroban after each stage is completed. Use the diagrams as a guide to check accuracy.

THE DIAGRAMS:

Problem: Find the square root of 2





With 7 decimal places in the answer we'll stop.

Following Prof. Kato's technique multiply the remainder 88,208,875 by 2 to equal 176,417,750.

The answer to the problem is **1.4142135** on rods **B** through **I** with a remainder of 176,417,750

18 March, 2006

On a personal note, writing this page 3 months ago required me to seriously examine Prof. Kato's technique. In order to illustrate it I really had to understand it. Since then I've become even more comfortable. I've benefited by routinely practicing using random examples of varying size and typically solving the problem to five or six decimal places. The technique works extremely well and is very efficient. It's well worth taking the time to understand it. In my opinion there is no better method. - TPH March 2006

- [Prof. Kato's Square Root Page](#)
- [Advanced Abacus Techniques](#)